

SPECTRAL EFFICIENCY BOUNDS FOR ADAPTIVE CODED MODULATION WITH OUTAGE PROBABILITY CONSTRAINTS AND IMPERFECT CHANNEL PREDICTION

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ABSTRACT

Adaptive coding and modulation (ACM) is a technique designed to combat the temporal variations of (slow) multipath fading channels. The main objective of an ACM scheme is to improve the spectral efficiency (SE) while accommodating a predefined bit error rate. The *average spectral efficiency* of an ACM scheme can be measured against the channel capacity for idealized conditions such as perfect knowledge of future states of the communication channel, the ability to instantly change channel code when the channel quality changes, and use of infinitely many channel codes of arbitrary length. Clearly, these conditions do not hold for a practical system. In this paper we shall present upper bounds on the SE of an ACM scheme that take into account imperfect knowledge of the channel state information, and a limited number of codecs. These bounds are also found for a given constant of the outage probability.

I. INTRODUCTION

Adaptive coded modulation (ACM) schemes utilize a set of channel codes and modulation techniques (“*codecs*”) with different spectral efficiencies (SEs) [bits/s/Hz] to transmit information on flat fading channels [1, 2, 3]. The bit error rate (BER) versus channel signal-to-noise ratio (CSNR) for each codec on an additive white Gaussian noise (AWGN) channel must be known. Then a CSNR threshold is defined for each codec such that it guarantees a BER below a certain target BER, denoted BER_0 , when the CSNR is above the CSNR threshold. The *channel state information* (CSI) at a future time instant is obtained by predicting the future CSNR at the receiver. The CSI is communicated to the transmitter via a separate feedback channel. The transmitter uses this information to select the codec with the highest SE, among the available codecs, that satisfies the BER demand. The overall transmission scheme will have a low SE when the predicted CSNR is low and a high SE when the predicted CSNR is high. The *average SE* (ASE) of an ACM system is found by averaging the available SE over the fading distribution.

Assuming perfect prediction of future CSNR values and assuming the fading to be approximately constant during a given time interval, a simple *codec selection strategy* (CSS) can be found by employing the CSNR thresholds discussed in the previous paragraph as *switching thresholds*. In practice however, the channel prediction is not perfect. The system can be made more robust towards channel prediction errors by increasing the switching thresholds so as to be more conservative in the choice of codecs.

The performance in terms of ASE can be compared to

channel capacity, but since a practical ACM scheme employs a limited number of codecs a more realistic reference is the optimal *maximum ASE for ACM* (MASA) [4]. In [4] the optimal MASA was found for the case of perfect channel prediction. This result can be used as an ultimate upper bound on the ASE of an ACM scheme with N codecs.

In this paper we shall also obtain bounds for the ASE of an ACM scheme with N component codecs, but now imperfect channel prediction is also taken into account, under the requirement that the average BER of each codec is less than BER_0 . The resulting MASA (in [4]) may result in a high outage probability. The method is therefore extended to optimize the MASA under an outage constraint.

II. SYSTEM MODEL

A. The communication channel

We shall consider the system in Figure 1. The communication channel under consideration is a time-varying, frequency-flat, slow multipath fading channel (MFC). In the complex baseband model of the MFC the received signal can be written as $y(t) = z(t) \cdot x(t) + n(t)$, where $x(t)$ is the transmitted complex-valued symbol, $n(t)$ is complex-valued AWGN, and $z(t)$ is the complex fading gain. The instantaneous and average CSNR are defined as

$$\gamma(t) = \frac{|z(t)|^2 \cdot P}{N_0 B} \quad (1)$$

and

$$\bar{\gamma} = E[\gamma(t)] = \frac{\Omega_p \cdot P}{N_0 B}, \quad (2)$$

respectively, where P [W] is the average transmit power, N_0 [W/Hz] is the one-sided noise power spectral density, B [Hz] is the one-sided transmission bandwidth, and $\Omega_p = E[|z(t)|^2]$ is the average power gain. We shall assume that the magnitude of the received complex fading envelope $|z(t)|$ has a Rayleigh distribution [5]. Then, the CSNR has an exponential distribution with expectation $\bar{\gamma}$:

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\frac{\gamma}{\bar{\gamma}}}. \quad (3)$$

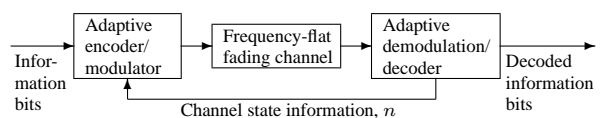


Figure 1: System model.

B. Adaptive coding and modulation

The system in Figure 1 consists of a transmitter and receiver employing N component codecs, the communication channel, and a separate return channel which is assumed free of errors, but with a nonzero delay. The codecs are indexed by $n \in \{1, 2, \dots, N\}$ and designed for AWGN channels at different CSNRs. The range of possible CSNR values on the MFC, $\gamma \in [0, \infty)$, is divided into $N + 1$ CSNR regions by $N + 2$ CSNR thresholds denoted γ_n as outlined in Figure 2, with $\gamma_0 = 0$ and $\gamma_{N+1} = \infty$. The CSNR falls into CSNR region n when $\gamma_n \leq \gamma < \gamma_{n+1}$, $n \geq 0$. The CSNR thresholds $\gamma_1, \gamma_2, \dots, \gamma_N$ are based on the BER performance for each of the N codecs on AWGN channels, and are selected such that the BER of codec n is less than or equal to BER_0 when $\gamma \geq \gamma_n$, $n \geq 0$ (we refer to [3] for details). The fundamental idea of an ACM scheme is that when $\gamma \in [\gamma_n, \gamma_{n+1})$, codec n is the codec with highest SE that guarantees a BER below the target BER. Thus, it is assumed that $R_1 < R_2 < \dots < R_N$, where R_n denotes the SE of codec n . When the CSNR falls in the zeroth region, the *outage* region, none of the employed codecs can guarantee a BER below the target BER. The ASE of the ACM scheme is defined as [1]

$$\text{ASE} = \sum_{n=1}^N R_n \cdot P_n \text{ [bits/s/Hz]}, \quad (4)$$

where P_n is the probability of codec n being used.

The CSI, the index of the selected codec, is obtained by channel prediction. After the prediction, the appropriate codec index n selected by the CSS is transmitted from the receiver to the transmitter on the separate return channel. When the CSI changes, the transmitter and receiver adapt—i.e. perform a codec update—to maximize the SE.

In order to provide the ACM scheme with information on the fading envelope we shall assume that a maximum a posteriori (MAP) optimal predictor is employed [6]. In this case the predicted CSNR follows an exponential distribution with expectation $\rho\bar{\gamma}$ [6], where ρ is the *normalized correlation* between the actual and predicted CSNR. In [6] the performance of a MAP-optimal predictor was investigated, and it was found that ρ is a function of $\bar{\gamma}$ (as well as of the maximal Doppler spread and the return channel delay). The general bounds obtained in this paper depends on both the correlation ρ and $\bar{\gamma}$.

C. Codec selection strategies

Since the actual CSNR is unknown at the receiver we define a CSS as a set new set of thresholds that are used to select a codec based on the predicted CSNR. The switching thresholds are denoted $\{s_n\}_{n=0}^{N+1}$ with $s_0 = 0$ and $s_{N+1} = \infty$. Codec n is selected when $\hat{\gamma}$ falls in *switching region* n , defined as $[s_n, s_{n+1})$. The probability of selecting codec n

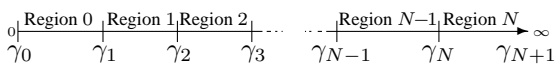


Figure 2: CSNR thresholds and the $N + 1$ CSNR regions corresponding to N codecs.

can then be written as

$$P_n = \int_{s_n}^{s_{n+1}} p_{\bar{\gamma}}(\hat{\gamma}) d\hat{\gamma} = \int_{s_n}^{s_{n+1}} \frac{1}{\rho\bar{\gamma}} e^{-\frac{\hat{\gamma}}{\rho\bar{\gamma}}} d\hat{\gamma}. \quad (5)$$

When $\hat{\gamma} \in [s_0, s_1)$ the system experiences an outage and only pilot information is transmitted. The probability of outage is thus given by P_0 (and is found by substituting $n = 0$ into (5)).

When perfect prediction is assumed ($\rho = 1$), the CSNR thresholds in Figure 2 can be used as switching thresholds. That is, $s_n = \gamma_n$ for $n \in \{1, \dots, N\}$. As the normalized correlation ρ is reduced (corresponding to either a lower $\bar{\gamma}$, a larger return channel delay, or a higher terminal velocity) there is an increasing probability of mismatch between the predicted and actual CSNR. This will result in an increased BER, since the actual CSNR sometimes will fall into a lower indexed region than the predicted CSNR. Although this cannot be completely avoided with any CSS it is desirable to control the probability of this event. It is then natural to demand

$$P(\gamma < \gamma_n | \hat{\gamma} = s_n) = \int_0^{\gamma_n} p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma} = s_n) d\gamma = \epsilon, \quad (6)$$

where ϵ is some (small) constant chosen by the designer. Thus, by increasing the switching thresholds, $\{s_n\}_{n=1}^N$, to obtain a certain desired (sufficiently small) ϵ the probability of a codec mismatch can be reduced in a more controlled manner. As the predicted CSNR increases the probability of a codec mismatch is reduced, and thus, if s_n is chosen to fulfill (6),

$$P(\gamma < \gamma_n | \hat{\gamma} \geq s_n) \leq \epsilon. \quad (7)$$

From [7] we know that the probability of the complementary event is given as

$$1 - \epsilon = P(\gamma > \gamma_n | \hat{\gamma} = s_n) = Q\left(\frac{\sqrt{s_n}}{\omega}, \frac{\sqrt{\gamma_n}}{\omega}\right), \quad (8)$$

where $\omega = \sqrt{\bar{\gamma}(1-\rho)}/2$ and

$$Q(a, b) = \int_b^{\infty} x I_0(ax) e^{-\frac{1}{2}(a^2+x^2)} dx \quad (9)$$

is the Marcum-Q function [8]. The switching thresholds can now be found by solving (8) with respect to every s_n , $n \in \{1, 2, \dots, N\}$ for given values of $\bar{\gamma}$, ρ , $\{\gamma_n\}_{n=1}^N$, and a chosen $\epsilon \in (0, 1)$. Note that a higher level of protection against the event of choosing a code with too high SE means a smaller value for ϵ .

III. AN UPPER BOUND ON THE ASE OF ACM

The *maximum average spectral efficiency* (MASE) for the Rayleigh fading channel is the expected channel capacity in the Shannon sense, i.e., the ultimate upper theoretical limit for the ASE with constant power, and is given as [9, Eq. (23)]

$$\text{MASE}(\bar{\gamma}) = -e^{\frac{1}{\bar{\gamma}}} \log_2(e) E_i\left(-\frac{1}{\bar{\gamma}}\right) \text{ [bits/s/Hz]}. \quad (10)$$

Here $E_i(\cdot)$ is the *exponential-integral function* [10, Equation 8.211.1]

$$E_i(x) = -\int_{-x}^{\infty} \frac{e^{-t}}{t} dt \quad \text{for } x < 0. \quad (11)$$

The MASE can only be achieved if infinitely many codecs are employed and perfect prediction is assumed.

The MASA presented in [4] is obtained when each of the $N < \infty$ codecs employed have the maximum SE in the corresponding CSNR regions. Following the idea in [4] we assume that we possess N codecs that achieve capacity on AWGN channels, and that the fading is constant between two successive codec updates. The SE of a codec achieving capacity at $\gamma = \gamma_k$ is given as

$$R_k = C(\gamma_k) = \log_2(1 + \gamma_k) \text{ [bits/s/Hz]}, \quad (12)$$

where $C(\gamma)$ denotes the channel capacity of an AWGN channel with CSNR γ . Each of the codecs employed are assumed to perform with BER = 0 when the CSNR is above the corresponding CSNR threshold. When the CSNR is below the CSNR threshold the codecs have BER $\leq \frac{1}{2}$. To obtain the MASA the N codecs must be chosen such that they achieve capacity at the lower boundary of the corresponding CSNR region (in order to have zero BER in the entire region). The SE of codec n becomes $R_n = \log_2(1 + \gamma_n)$ and the MASA

$$\text{MASA} = \sum_{n=1}^N \log_2(1 + \gamma_n) P_n \quad (13)$$

is an upper bound on the ASE.

The objective of this paper is to obtain the N switching thresholds $\{s_n\}_{n=1}^N$ (and the corresponding CSNR thresholds) that optimize the MASA. The first step towards obtaining these thresholds is to choose a value for ϵ for the capacity achieving codecs. Then, for given values of ρ and $\bar{\gamma}$ the relationship between the switching thresholds and the corresponding CSNR thresholds are known and the MASA can be optimized.

A. Optimal switching thresholds

Since codec n is only used when $\hat{\gamma} \in [s_n, s_{n+1})$ the BER of codec n averaged over all CSNRs can be written as

$$\begin{aligned} \overline{\text{BER}}_n &= \int_0^{\gamma_n} \overbrace{\text{BER}_n(\gamma)}^{\leq \frac{1}{2}} p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) d\gamma \\ &\quad + \int_{\gamma_n}^{\infty} \overbrace{\text{BER}_n(\gamma)}^{=0} p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) d\gamma \\ &\leq \frac{1}{2} \int_0^{\gamma_n} p_{\gamma|\hat{\gamma}}(\gamma|\hat{\gamma}) d\gamma \leq \frac{1}{2} \epsilon. \end{aligned} \quad (14)$$

Since the average BER of codec n should be less than BER₀ we demand

$$\overline{\text{BER}}_n \leq \frac{\epsilon}{2} \leq \text{BER}_0 \Rightarrow \epsilon \leq 2 \cdot \text{BER}_0. \quad (15)$$

The expression for the MASA in (13) contains both γ_n and s_n (after inserting (5)). The switching thresholds $\{s_n\}_{n=1}^N$, which are the region boundaries for the predicted CSNR, are related to the CSNR thresholds, which are the region boundaries for the actual CSNR, through the Marcum-Q function in (8). To the authors' knowledge there does not exist a closed form expression for the inverse of the Marcum-Q function. But the inverse of one

of the arguments can be obtained, with a given accuracy, using e.g. Ridders' method [11]. That is, the inverse of the complementary Marcum-Q function, $1 - Q(a, b)$, with respect to its first and second argument can be defined as $a = q_a(b, \epsilon)$ and $b = q_b(a, \epsilon)$, respectively. Then the inverse of $Q\left(\frac{\sqrt{s_n}}{\omega}, \frac{\sqrt{\gamma_n}}{\omega}\right)$ with respect to s_n can then be written as (for $n \in \{1, 2, \dots, N\}$)

$$\gamma_n(\epsilon) = \Psi_s(s_n, \epsilon) = \begin{cases} \left(\omega \cdot q_b\left(\frac{\sqrt{s_n}}{\omega}, \epsilon\right)\right)^2 & \rho < 1 \\ s_n & \rho = 1. \end{cases} \quad (16)$$

The MASA in (13) can now be written as

$$\begin{aligned} \text{MASA}(\epsilon) &= \frac{1}{\ln(2)} \sum_{n=1}^N \ln(1 + \Psi_s(s_n, \epsilon)) \\ &\quad \times \int_{s_n}^{s_{n+1}} p_{\hat{\gamma}}(\hat{\gamma}) d\hat{\gamma}. \end{aligned} \quad (17)$$

The expressions for both the MASA and γ_n (obtained from s_n) are functions of ϵ . In the following we shall assume that the upper limit on ϵ is used: $\epsilon = 2 \cdot \text{BER}_0$, and to simplify notation we do not include the ϵ in the expressions throughout the rest of the paper.

B. Theoretical upper bounds

The MASA is a function of the switching thresholds $\{s_n\}_{n=1}^N$. Thus, optimizing the MASA is done by first obtaining the gradient of the MASA with respect to the switching thresholds. Setting the gradient equal to zero results in N equations that can be used to find expressions for the optimal switching thresholds. It might be desirable to obtain the optimal MASA under a constraint on the probability of outage, e.g. if the quality of service of a communication system is defined in terms of continuous transmission. This can be achieved by introducing a Lagrange multiplier, λ , into the optimization procedure as follows

$$\nabla_{\{s_n\}_{n=1}^N} (\text{MASA} + \lambda \cdot P_{\text{out}}) = \mathbf{0}. \quad (18)$$

Setting $\lambda = 0$, the solution to (18) yields the optimal switching thresholds that produce the optimal MASA, while $\lambda \neq 0$ yields suboptimal switching thresholds that satisfy the demand on probability of outage, $P_0 = P_{\text{out}}$.

Solving (18) (and multiplying the result with $\ln(2)$) produces the following set of equations:

$$\begin{bmatrix} p_{\hat{\gamma}}(s_1) (\ln(2) \lambda - \ln(1 + \Psi_s(s_1))) \\ \quad + \frac{\frac{\delta}{\delta s_1} \Psi_s(s_1)}{1 + \Psi_s(s_1)} P_1 \\ \\ \ln\left(\frac{1 + \Psi_s(s_1)}{1 + \Psi_s(s_2)}\right) p_{\hat{\gamma}}(s_2) + \frac{\frac{\delta}{\delta s_2} \Psi_s(s_2)}{1 + \Psi_s(s_2)} P_2 \\ \\ \vdots \\ \\ \ln\left(\frac{1 + \Psi_s(s_{n-1})}{1 + \Psi_s(s_n)}\right) p_{\hat{\gamma}}(s_n) + \frac{\frac{\delta}{\delta s_n} \Psi_s(s_n)}{1 + \Psi_s(s_n)} P_n \\ \\ \vdots \\ \\ \ln\left(\frac{1 + \Psi_s(s_{N-1})}{1 + \Psi_s(s_N)}\right) p_{\hat{\gamma}}(s_N) + \frac{\frac{\delta}{\delta s_N} \Psi_s(s_N)}{1 + \Psi_s(s_N)} P_N \end{bmatrix} = \mathbf{0} \quad (19)$$

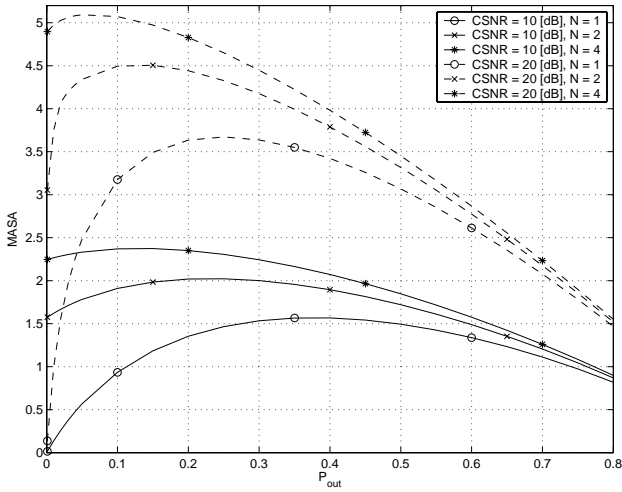


Figure 3: Optimal MASA as a function of P_{out} for $\bar{\gamma} \in \{10, 20\}$ [dB], $N \in \{1, 2, 4\}$, and $\rho = 1$ (perfect prediction is assumed).

Then, by setting $P_0 = P_{\text{out}}$, solving (5) (with $n = 0$) for s_1 , and solving equation n in (19) for s_{n+1} (note: to find the expression for s_{n+1} the expression for P_n , in (5), must be substituted into (19)), the following three equations are obtained:

$$s_1 = -\ln(1 - P_{\text{out}}) \bar{\gamma} \rho \quad (20)$$

$$s_2 = s_1 - \bar{\gamma} \rho \times \ln \left(1 - \frac{1 + \Psi_s(s_1)}{\bar{\gamma} \rho \frac{\delta}{\delta s_1} \Psi_s(s_1)} \ln \left(\frac{1 + \Psi_s(s_1)}{2^\lambda} \right) \right) \quad (21)$$

$$s_n = s_{n-1} - \bar{\gamma} \rho \times \ln \left(1 - \frac{1 + \Psi_s(s_{n-1})}{\bar{\gamma} \rho \frac{\delta}{\delta s_{n-1}} \Psi_s(s_{n-1})} \ln \left(\frac{1 + \Psi_s(s_{n-1})}{1 + \Psi_s(s_{n-2})} \right) \right) \quad (22)$$

for $n \in \{3, 4, \dots, N\}$.

That is, every switching threshold s_n (for $n > 1$) can be found recursively, and as a result the MASA is a function of s_1 and λ . When $\lambda = 0$ the MASA can be optimized by first obtaining the optimal s_1 . This is done by searching through all possible values of s_1 (or equivalently $P_{\text{out}} \in \langle 0, 1 \rangle$). When the outage constraint is employed the value of s_1 is given from (20), and a similar search must be made through all possible values of λ .

IV. RESULTS AND DISCUSSION

In Figure 3 the suboptimal MASA is plotted as a function of P_{out} for $\bar{\gamma} \in \{10, 20\}$ [dB], $\rho = 1$ (that is, perfect prediction is assumed), and $N \in \{1, 2, 4\}$. For each of the curves, the maximum value corresponds to the optimal MASA ($\lambda = 0$). As can be seen from the results, the MASA increases with both N and $\bar{\gamma}$. That is, for higher values of N an ACM system can use codecs with higher SEs, resulting in an overall increased ASE. Increasing $\bar{\gamma}$

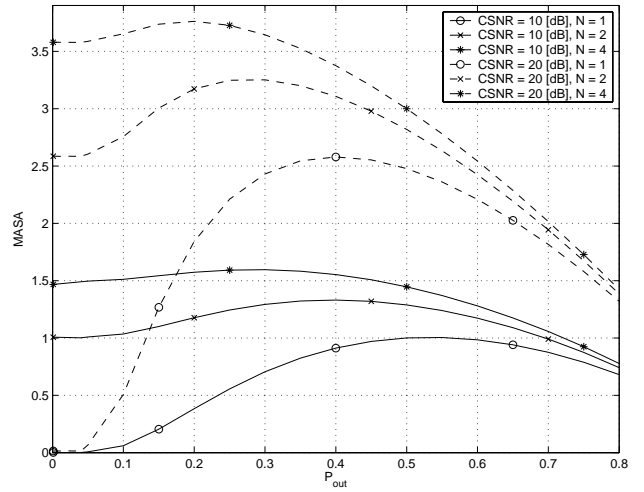


Figure 4: Optimal MASA as a function of P_{out} for $\bar{\gamma} \in \{10, 20\}$ [dB], $N \in \{1, 2, 4\}$, and $\rho = 0.99$.

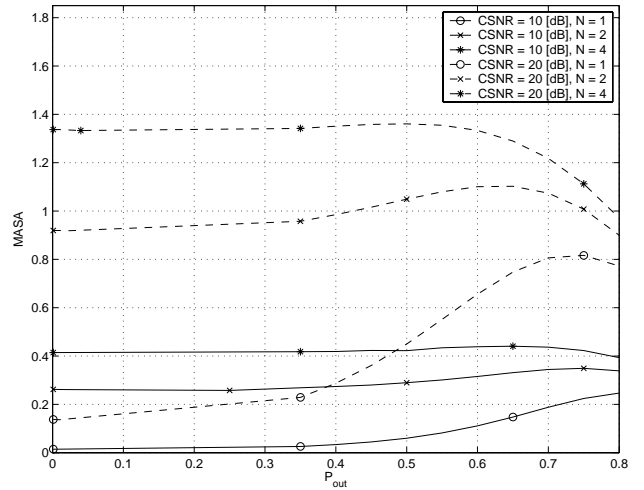


Figure 5: Optimal MASA as a function of P_{out} for $\bar{\gamma} \in \{10, 20\}$ [dB], $N \in \{1, 2, 4\}$, and $\rho = 0.90$.

increases the probability of actually having a higher instantaneously CSNR and thus codecs with higher SEs can be used by an ACM scheme and the resulting optimal switching thresholds increase.

Increasing the value of N reduces the outage probability that result in an optimal MASA. That is, increasing N also allows an ACM scheme to support codecs with lower SEs, and thus the switching threshold of the first codec s_1 is reduced producing a lower P_{out} . In addition it can be seen from the figure that the MASA is less sensitive to changes in the P_{out} as N increases. Thus, for a system employing a high number of codecs introducing an outage demand does not reduce the MASA as much as for a system employing a low number of codecs. Increasing the average CSNR increases all the optimal thresholds since the probability of predicting a higher CSNR increases. Thus, s_1 increases with $\bar{\gamma}$. Then, reducing s_1 according to a decrease in P_{out} results in a larger decrease as the average CSNR increases.

In Figures 4 and 5 the correlation was reduced to $\rho = 0.99$ and $\rho = 0.90$, respectively. (As an example, tak-

ing a carrier frequency $f_c = 2$ GHz terminal velocity $v = 30$ m/s, $\bar{\gamma} = 20$ [dB], a return channel delay of $\tau = 500$ μ s, and a MAP-optimal predictor of order $K = 1000$ and pilot symbol spacing $L = 10$, the normalized correlation will be $\rho \cong 0.995$. At $\bar{\gamma} = 10$ [dB], $\rho \cong 0.975$). Comparing these figures and Figure 3 it can easily be seen that the suboptimal MASA decreases with decreased ρ . Also, the value of P_{out} that results in the optimal MASA increases with decreased ρ . That is, as the correlation is reduced the probability of the predicted CSNR falling in a higher indexed CSNR region than the actual CSNR increases, and thus all the optimal switching thresholds must be increased when the correlation is reduced. The sensitivity to changes in P_{out} is less for lower values of ρ . This results from the fact that although the switching thresholds increase the CSNR thresholds may decrease and thus so does the SE of each codec and thus the overall MASA. Then by reducing s_1 according to a reduced P_{out} the SE of the first code is already very low and does not contribute much to the overall ASE.

Also, by comparing the figures it can be seen that the optimal MASA increases more for increasing values of N when $\bar{\gamma}$ increases. That is, when the channel conditions are good the increased gain in terms of SE by adding another codec is higher than for the case of a severely noisy and fast varying channel. Since the MASA is not sensitive to an outage constraint for low values of $\bar{\gamma}$ it might therefore be sufficient to employ only a small number of codecs.

When the normalized correlation ρ is severely reduced, the gain from adding another codec is not very high. Furthermore, the optimal MASA for an ACM scheme is severely reduced. It can therefore be concluded that ACM should only be employed on systems where the normalized correlation is sufficiently high.

V. CONCLUDING REMARKS

We have found bounds for the ASE of a rate adaptive communication scheme employing a limited number of codecs when imperfect channel knowledge is taken into account. The correlation between predicted and actual values of the fading envelope affects the bound in the sense that the bound is reduced as the correlation is reduced. Likewise, the bound increases with increasing average CSNR. It should be noted that the correlation between predicted and actual correlation itself depends on the average CSNR as well as on the choice and complexity of the predictor employed by the ACM scheme.

The bound was found by first presenting a unified tool for optimization of switching thresholds in rate adaptive communication. This tool also includes the possibility of introducing an upper limit on the probability of outage, since optimizing the switching thresholds may produce a very high outage probability. The results presented here show that as long as the average CSNR is sufficiently high there is a substantial gain in having more than two codecs in an ACM scheme.

By introducing an outage demand the optimal MASA is reduced. However, the reduction in suboptimal MASA is highest for a low number of codecs. Thus, for a system with a very strict demand on the outage probability it is beneficial

to use an ACM scheme with more than two codecs.

The results presented also shows that employing ACM should only be considered when the normalized correlation ρ is sufficiently high in the sense that there is a substantial gain in terms of SE by using more than one codec.

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